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Some Aspects Of Magnetohydrodynamics Flow on Flat plate with Suction

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ABSTRACT— A simple solution for the hydrodynamic linear equations for two dimensional flow, past a flat plate, in the presence of constant suction, have been obtained when the free stream follows exponential (increasing or decreasing) law. Asymptotic behavior of the transient motion has been discussed in detail which is particularly important in the decreasing case. Skin-friction follows exponentially increasing or decreasing laws respectively, with the parameter n, in the two cases.

2000 MATHEMATICS SUBJECT CLASSIFICATION : 76W05, 83C22, 46E30 or 22E43

KEYWORDS— **Lorentz force, Skin friction, Maxwell's stresses, transient motion, Hartmann number(M)**

I. INTRODUCTION

- Magnetic fields influence many natural and man-made flows. MHD is concerned with the mutual interaction of fluid flow and magnetic fields.

- As in certain kind of differential rotation can cause intense stretching and twisting of magnetic fields and may be a component of the process by which the earth maintains its magnetic field. There is incorporation of the Lorentz force into the Navier Stokes equations and considering some of the elementary and immediate consequences of this force.

- The combined magnetic field (imposed pulse induced) interacts with the induced current density , to give rise to a Lorentz force J X B

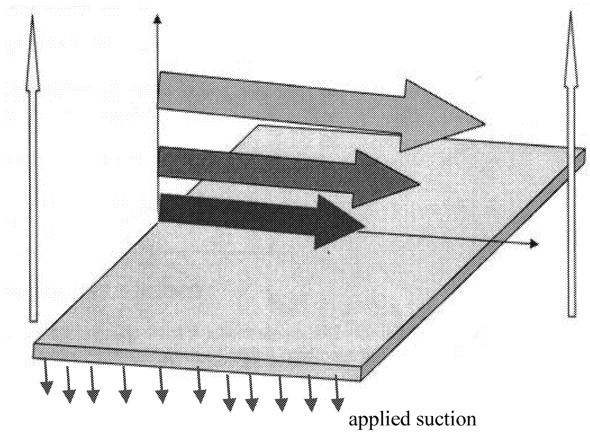
- Our research work focuses on the simple solution for the hydrodynamics linear equations which have been obtained for two dimensional flow, in the presence of constant suction.

II. Formulation of the Problem

The Governing equations of Fluid Flow in vector form as

$$\frac{Dv}{Dt} = -\frac{1}{\rho}\nabla p + \nu\nabla^2 v + \frac{1}{\rho}\overline{J} X \overline{B} \quad \text{-------------------------- (A)}$$

Where v- Velocity of the fluid
p- the pressure
ρ- the density
ν- the kinematic viscosity
J- the current density
B-the electromagnetic induction.



The flow with constant suction of a viscous incompressible and electrically conducting fluid past an infinite porous plate in the presence of a uniform transverse magnetic field, when the free –stream velocity follows exponential law has been considered.

III OBJECTIVES OF THE PROPOSED RESEARCH WORK

To study as to how much velocity field is affected with or without “suction and constant-magnetic field”. To consider these cases Laplace Transform has been taken. This is particularly important in the decreasing case where there is Phase to be determined. As it could happen, in the exponentially decreasing case, that the disturbance caused by the initial conditions always predominates. The later solution also helps us to understand how the transient motion dies out.

IV LITERATURE SURVEY

- The solution is interesting for its simple and concise form.
- [1] Previously Pandey K.S. has discussed the non- magnet case in studying the nature of skin friction .He has shown that there is no back flow near the wall.
- [2] Suryaprakasha Rao has discussed the flow with constant suction of a viscous incompressible and electrically conducting fluid past an infinite porous plate in the presence of a uniform transverse magnetic field, when the free-stream velocity oscillates in magnitude but not in direction .
- [3] I.Pop studied about the suction velocity which varies periodically with time on the unsteady free convection flow past a vertical porous plate
- Firstly without considering the initial conditions we have attempted to solve the hydro-magnetic equation when the free-stream is of the form $U_0(1+\in e^{\pm nt})$ where n is a positive constant, with the help of boundary conditions
  U = 0 at y = 0, u → U, (free-stream) as y → ∞. But each of our solutions thus obtained, give certain value of the velocity at time t = 0.
- The motion of two dimensional incompressible electrically conducting fluid flow along an infinite porous wall is considered. The flow is independent of the distance parallel to the wall and the suction velocity v'_0 , normal to the wall, is directed towards the wall and is constant. The X−axis is taken along the wall, Y−axis, normal to the wall.

In case of the infinite flat plate the continuity and momentum equations are [10]

$$\frac{\partial v'}{\partial y'} = 0 \ldots\ldots\ldots\ldots\ldots\ldots (1)$$

$$\frac{\partial v'}{\partial t'} = -\frac{1}{\rho'}\frac{\partial p'}{\partial y'} \ldots\ldots\ldots\ldots (2)$$

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = -\frac{1}{\rho'}\frac{\partial p'}{\partial x'} + \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma'}{\rho'}B'^2_0 u' \ldots\ldots\ldots\ldots \quad (3)$$

B'_0 --is the external magnetic field,
σ' – electrical conductivity of the field,
ρ' - density,
p'_0 - pressure,
v' - the kinematic coefficient of viscosity and
U'(t) the free stream velocity. Dashes denote the dimensional quantities.

From equation (1) it is clear that v' is function of time only. Consideration is further restricted to the case when v_0 equals to a negative constant ($-v'_0$) from which it follows that p'_0 is independent of y'. Consequently,

$$-\frac{1}{\rho'}\frac{\partial p'}{\partial x'} \text{ is equal to } \frac{\partial U'}{\partial t'} + \frac{\sigma'}{\rho'} B'^2_0\, U \quad (4)$$

Thus equation (3) becomes,

$$\frac{\partial u'}{\partial t'} - v'_0\frac{\partial u'}{\partial y'} = \frac{\partial U'}{\partial t'} + v'\frac{\partial^2 u'}{\partial y'^2} + \frac{\sigma'}{\rho'} B'^{\,2}_0 (U' - u') \quad (5)$$

V. WORK CARRIED OUT SO FAR

Firstly we consider the case when the velocity is subjected to the boundary conditions.

$u'_0 = 0$ at $y'_0 = 0$

$u' \rightarrow U'(t)$ for $y' \rightarrow \infty$

Secondly we will discuss the case like u = 0 at t = 0, etc. In both the cases for simplicity we assume
all physical properties of fluid such as viscosity, electrical conductivity and magnetic permeability etc. are constants. Further assumption is also made that magnetic Prandtl number of the induced electrical current do not affect the magnetic field., i.e. the applied magnetic field is essentially undisturbed by the flow field. In equation (3) all the electromagnetic quantities are measured in electromagnetic system of units. From equation (1) it is clear that v' is function of time only. Consideration is further restricted to the case when v_0 equals to a negative constant $(-v'_0)$ from which it follows that
p'_0 is independent of y'. Consequently,

From Maxwell's stresses in equation (A)

$$\because J \times B = (B \cdot \nabla)\left[\frac{B}{\mu}\right] - \nabla\left[\frac{B^2}{2\mu}\right]$$

as smaller value of $\nabla[\frac{B^2}{2\mu}]$

$$\therefore J \times B = (B \cdot \nabla)\left[\frac{B}{\mu}\right]$$

$$= (B_1 i + B_2 j + B_3 k) \cdot (i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z})[\frac{B}{\mu}]$$

$$= \frac{1}{\mu}(B_1\frac{\partial}{\partial x} + B_2\frac{\partial}{\partial y} + B_3\frac{\partial}{\partial z})(B_1 i + B_2 j + B_3 k)$$

For $B_1 = B_2 = 0 ; B_3 = B_0$

$$= \frac{1}{\mu}\left(B_0\frac{\partial}{\partial z}B_0 k\right)$$

$$\because J \times B = \frac{1}{\mu}\left(B_0{}^2\frac{\partial}{\partial z}k\right)$$

Substitute in (A)

$$\frac{Dv}{Dt} = -\frac{1}{\rho}\nabla p + \nu\nabla^2 v + \frac{1}{\rho\mu}\left(B_0{}^2\frac{\partial}{\partial z}k\right)$$

subject to conditions,

$u' = 0$ at $y' = 0$

$u' \rightarrow U'(t)$ for $y' \rightarrow \infty$

Non-dimensional quantities may be introduced in the following manner,

$$y = \frac{y'|v'_0|}{\nu'} \quad ; \quad t = \frac{v'_0\, t'}{4\nu'} \quad ; \quad \eta = \frac{4\nu'\eta'}{v_0'^2}$$

$$u = \frac{u'}{U'_0} \;;\quad U = \frac{U'}{U'_0} \;;\quad M = \frac{4\nu'\sigma' B'^2_0}{v'^2_0} \tag{6}$$

where U'_0 is the reference velocity. Now the equation (5) is,

$$\frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y} - \frac{1}{4}\frac{\partial u}{\partial t} = -\frac{1}{4}\frac{\partial U}{\partial t} - \frac{M}{4}(U - u) \tag{7}$$

subject to the conditions,

$u = 0$ at $y = 0$,

$u \to U(t)$ for $y \to \infty$ (8)

VI. RESULTS AND DISCUSSION

Now a particular case when the free-stream follows exponential law, i.e. U is of the form $(1 + \epsilon e^{\pm nt})$, where n is a positive constant, will be considered. It may be mentioned here that in three cases time span should be small to make it physically significant. Here ϵ is an arbitrary constant. Thus

$$U = 1 + \epsilon e^{\pm nt}, \tag{9}$$

Therefore let us suppose that,

$$u = 1 + \epsilon e^{\pm nt} - f_1(y) - \epsilon e^{\pm nt} f_2(y), \tag{10}$$

where $f_1(y)$ and $f_2(y)$ are functions of y to be determined later on. Substituting in equation (7) and comparing the term of the same family and neglecting the second power of ϵ , for simplicity, we get,

$$\frac{d^2 f_1}{dy^2} + \frac{df_1}{dy} - \frac{M}{4} f_1 = 0 \tag{11}$$

$$\frac{d^2 f_2}{dy^2} + \frac{df_2}{dy} - \frac{\pm n + M}{4} f_2 = 0 \tag{12}$$

subject to the conditions,

$y = 0$ then $f_1 = f_2 = 1$,

$y \to \infty$ then $f_1 = f_2 \to 0$ (13)

The solution of equation (11) and (12) satisfying (13) are

$f_1 = e^{-\alpha y}$ where $\alpha = \frac{1+\sqrt{1+M}}{2}$,

and $f_2=e^{-\beta y}$ where $\beta = \frac{1+\sqrt{1+M\pm n}}{2}$ (14)

Hence the expression for the velocity and skin-friction in the boundary layer are,

$$u(y,t)=1+\epsilon e^{\pm nt}-e^{-\alpha y}-\epsilon e^{\pm nt-\beta y} \quad (15)$$

One may find the expression of skin-friction as,

$$\tau_0 = \frac{\tau'_0}{U'|v'_0|} = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{1+\sqrt{1+M}}{2} + \frac{1+\sqrt{1+M\pm n}}{2} \quad \epsilon e^{\pm nt} \quad (16)$$

which obviously follows exponentially increasing or decreasing laws, when the free stream follows exponentially increasing or decreasing laws respectively, with the parameter n.

For h = M +1 (say)

(i) h<|n|, then velocity expression (in the increasing case) to a good approximation may be written as,

$$u = 1 + \epsilon\, e^{\pm nt}-e^{-\alpha y} -\epsilon\, e^{\pm nt-y(\frac{1}{2}+\frac{1}{2}\sqrt{n}+\frac{1}{4}\frac{h}{\sqrt{n}}-\frac{1}{16}\frac{h^2}{n^{3/2}})}$$

If (ii) if h >|n|, then in both the cases,

$$u = 1 +\epsilon\, e^{\pm nt}-e^{-\alpha y} -\epsilon\, e^{\pm nt-y(\frac{1}{2}+\frac{\sqrt{h}}{2}\pm\frac{1}{4}\frac{n}{\sqrt{h}}\mp\frac{1}{16}\frac{n^2}{h^{3/2}})}$$

Figure 1 has been drawn by plotting the velocity distribution, u, against the y-axis (for exact and approximate values, h > n) for approximately increasing case. It shows the later to be in fair agreement with the exact one.

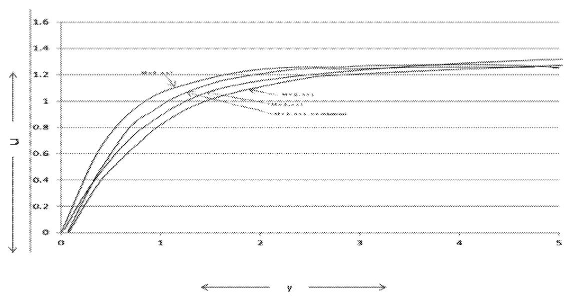


Fig.1 A graph of the velocity distribution u against y-values

Table of Fig 1 x axis and y axis values	
X- axis (values of y)	Y- axis(values of velocity distribution u)
M= 0, n = 1	
0	0.0
0.5	0.4
1	0.7
2	1
3	1.1
4	1.2

Table of Fig 1 x axis and y axis values	
X- axis(values of y)	Y- axis(values of velocity distribution u)
h > n; M= 2, n = 1	
0	0.0
0.5	0.43
1	0.73
2	1.1
3	1.19
4	1.2

Table of Fig 1 x axis and y axis values	
X- axis (values of y)	Y- axis(values of velocity distribution u)
M= 4, n = 1	
0	0.0
0.5	0.49
1	0.79
2	1.19
3	1.2
4	1.2

Table of Fig 1 x axis and y axis values	
X- axis(values of y)	Y- axis(values of velocity distribution u)
h < n ; M= 2, n = 1	
0	0.0
0.5	0.42
1	0.71
2	1.01
3	1.11
4	1.2

velocity distribution in the absence of magnetic field has also been shown in the figure, which indicates the rise in velocity in the magnetic case over the nonmagnetic and which is further enhanced with the rise in Hartmann number M.

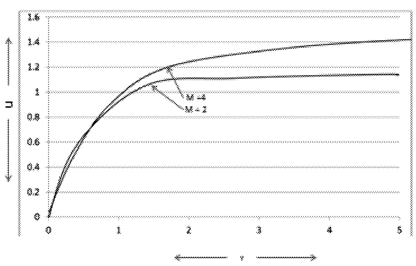


Fig. 2 velocity profile u has been plotted against y

Table of Fig 2x axis and y axis values	
X- axis(values of y)	Y- axis(values of velocity distribution u)
M= 2	
0	0.0
0.5	0.6
1.49	1.09
2.56	1.1
4.3	1.18
4.9	1.19

Table of Fig 2 x axis and y axis values	
X- axis(values of y)	Y- axis(values of velocity distribution u)
M= 4	
0	0.0
0.5	0.49
1	0.9
1.9	1.2
3.5	1.35
5	1.4

Fig.2 velocity profile u has been plotted against y for M = 2,4, in the exponentially decreasing case, both for exact and approximate values of velocity, which shows later to be in the best approximation with the exact values. The nature of the curve in the two increasing or decreasing cases are the same. Discussion on the decay of the transient motion; when the free stream follows exponentially decreasing law, one's concern is well founded that the disturbance caused by the initial conditions may predominate so that the previous

$$\left.\begin{array}{l} u = 0 \; y = 0 \\ u \to U, \quad y \to \infty \end{array}\right\} \text{For } t > 0$$
$$u = 0 \qquad \text{For } t \leq 0 \tag{17}$$

$$\text{If } \quad \bar{u}\,(p) = \int_0^\infty u(t)\, e^{-pt}\, dt \quad \text{and} \tag{18}$$

$$\bar{U}(p) = \int_0^\infty U(t)\, e^{-pt} dt \tag{19}$$

$$\text{Then } \bar{u}\,(p) = \bar{U}(p) - \bar{U}(p).\, e^{-\frac{1+\sqrt{1+p+M}}{2} y}$$

In particular, for U = $\in(1-e^{kt})$, (k may take positive or negative values)

$$\bar{U} = \in\left(\frac{1}{p} + \frac{1}{p-k}\right) \tag{20}$$

As in [4] this is a problem on dissipative system in finite region the solution is consists of series of negative exponentials in the time (which will be negligible for large values of the time) together, possibly, with constant, linear or quadratic,etc., terms arising from a pole of u(p) at the origin; these latter determine the asymptotic behavior of the function for large values of time. It often happens that the solution has linear asymptote and that only this is wanted; to find it, only the residue of $e^{pt}\bar{u}\,(p)$ at the origin need be calculated.

Asymptotic behavior of the transient motion has been discussed in detail which is particularly important in the decreasing case. Here we have attempted to solve the hydro-magnetic equation when the free-stream is of the form

$U_0(1+\in e^{\pm nt})$ where n is a positive constant.

Also help of Laplace Transform has been taken in particularly the decreasing case where there is phase, to be

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Also help of Laplace Transform has been taken in particularly the decreasing case where there is phase, to be determined. The later solution also helps us in discussing that how the transient motion dies out.

Determined the asymptotic behaviour of the function for large values of time. It often happens that the solution has linear asymptote and that only this is wanted. It is to be mentioned that to evaluate u completely, one is required to know the singularities of $\bar{u}(p)$, but to find its linear asymptote only the pole at origin is required.

It is to be mentioned that to evaluate 'u' completely, one is required to know the singularities of u(p), but to find its linear asymptote only the pole at origin is required.

Hence

$$u \sim \text{residue } [\bar{u}(p).\, e^{pt}]$$

$$= 1 - e^{-\frac{1+\sqrt{1+m}}{2} y}$$

this obviously, represents a linear asymptote for the velocity as $t \rightarrow \infty$.

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